INTEGRATION OF TWO DIFFERENTIAL FORMS.

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To integrate $x^m \sin x\,dx$ and $x^m \cos x\,dx$. Because $\int u dv = uv - \int v du$; let $u = \cos x$ and $dv = x^n dx$, then

$$\int x^n \cos x\,dx = \frac{1}{n+1} x^{n+1} \cos x + \frac{1}{n+1}\int x^{n+1} \sin x\,dx;$$

$$\therefore \int x^{n+1} \sin x\,dx = -x^{n+1} \cos x + (n+1)\int x^n \cos x\,dx. \qquad (1)$$

Again, let $u = \sin x$ and $dv = x^n dx$,

$$\int x^{n+1} \cos x\,dx = x^{n+1} \sin x - (n+1)\int x^n \sin x\,dx. \qquad (2)$$

Let $n = 0$ and substitute in (1) and (2),

$$\int x \sin x\,dx = -x \cos x - \sin x,$$

$$\int x \cos x\,dx = x \sin x - \cos x.$$

Let $n = 1$ and substitute in (1) and (2),

$$\begin{aligned} \int x^2 \sin x\,dx &= -x^2 \cos x + 2\int x \cos x\,dx \\ &= -x^2 \cos x + 2x \sin x - 2 \cos x, \\ \int x^2 \cos x\,dx &= x^2 \sin x - 2\int x \sin x\,dx \\ &= x^2 \sin x + 2x \cos x - 2 \sin x. \end{aligned}$$

Let $n = 2$ and substitute in (1) and (2),

$$\begin{aligned} \int x^3 \sin x\,dx &= -x^3 \cos x + 3\int x^2 \cos x\,dx \\ &= -x^3 \cos x + 3x^2 \sin x + 3.2x \cos x - 3.2 \sin x \\ \int x^3 \cos x\,dx &= x^3 \sin x - 3\int x^2 \sin x\,dx \\ &= x^3 \sin x + 3x^2 \cos x - 3.2x \sin x - 3.2 \cos x. \end{aligned}$$

Let $n = 3$ and substitute in (1) and (2), &c.

The laws of the series are thus determined, and may be expressed:

$$\begin{aligned} \int x^m \sin x\,dx &= -x^m \cos x + m x^{m-1} \sin x + m(m-1) x^{m-2} \cos x \\ &\qquad - m(m-1)(m-2) x^{m-3} \sin x - + + --, \\ \int x^m \cos x\,dx &= x^m \sin x + m x^{m-1} \cos x - m(m-1) x^{m-2} \sin x \\ &\qquad - m(m-1)(m-2) x^{m-3} \cos x + + --. \end{aligned}$$

[These integrals may be found at p. 265 of Hirsch's Integral Tables, but they were computed by Mr. Shack without suspecting that they were known forms.—Ed.]

NOTE ON THE SOLUTION OF PROB. 373.—We have received from Prof. H. T. Eddy, a brief and elegant solution of (373), in which several errors which occur in the published solution are pointed out and corrected. We have also received from Mr. Adcock the following corrections of his solution of that problem (see p. 55):